Workshop on Automorpic forms, representations of Lie groups and several complex variables

South Building of AMSS., CAS., Beijing

Hua Loo-Keng Center for Mathematical Sciences

July 4-14, 2018

July 5-6. Introductory lectures

July 5, 2018, 9:00-12:00. N202

Chengbo Zhu: Representation theory of real reductive groups: algebraic aspect.

Abstract: I will explain

(1) The basic theory of (g, K)-modules;

(2) A construction of the fundamental series;

(3) Unitarisable modules with nonzero (g, K)-cohomology;

(4) Invariants of Harish-Chandra modules.

References:

(1) Real Reductive Groups, I, by Nolan Wallach

(2) Associated varieties and unipotent representations, by David Vogan.

14:30 - 17:30. N202

Binyong Sun: Representation theory of real reductive groups: analytic aspect

Abstract: I will explain some basic notions and basic facts in representation theory of real reductive groups, emphasizing on the analytic aspect. Basic knowledges on topological spaces, smooth manifolds and Lie groups are required. I hope to cover the following materials:

1. Topological vector spaces and function spaces;
2. Representations and smooth representations;
3. Casselman-Wallach representations and smooth automorphic forms;
4. Generalized matrix coefficients and characters;
5. Discrete series representations, tempered representations and Langlands classification;
6. Generalized Whittaker models.

References:
N. R. Wallach: Real Reductive Groups, I, Academic Press Pure and Applied Mathematics, Boston, 132 (1988).
N. R. Wallach: Real Reductive Groups II, Academic Press, Boston, 1992.

July 6, 9:00-12:00. N202
Kai-Wen Lan: An example-based introduction to Shimura varieties
Abstract: I will start by explaining why Shimura varieties are natural generalizations of the more classical objects called modular curves, and give many examples with explicit descriptions of groups and symmetric domains. If time permits, I will also introduce their boundary components, and discuss about their models over rational numbers or even integers. The lectures will be for people who are not already familiar with these topics—for most of them, some willingness to see matrices larger than 2x2 ones should suffice. (I hope to allow simple factors of all possible types A, B, C, D, and E to show up if time permits, but it is not necessary to know beforehand what this means.)
References: Introductory lecture notes,
http://www.math.umn.edu/ kwlan/articles/intro-sh-ex.pdf

14:30 - 17:30. N202
Genkai Zhang: Complex analysis and representation theory related to Hermitian symmetric spaces.
Abstract: Hermtian symmetric spaces and their quotient spaces form an important class of complex manifolds and they appear in geometry as moduli spaces and in number theory. I'll start with their geometric/algebraic definitions and explain the following topics:

1. Harish-Chandra realization, Siegel domain realization, and related realization in projective spaces,
2. Bergman spaces, Bergman metric and kernels, the Hua-Kostant-Schmid decomposition and reproducing kernel expansion.
3. Introduction of general holomorphic discrete series generalizing Bergman spaces.
4. Plancherel formula on compact and non-compact Hermitian symmetric spaces.
References:
1. S. Helgason, Differential geometry, Lie groups and Symmetric spaces.
2. J. Faraut and A. Koranyi, Analysis on symmetric cones.

July 9-13. Workshop

July 9, 9:30-10:30, N202
Jing-Song Huang: Dirac operators, orbit method and unipotent representations.
Abstract: The method of coadjoint orbits for real reductive groups is divided into three steps in cooperation with the Jordan decomposition of a coadjoint orbit into hyperbolic part, elliptic part and nilpotent part. This is formulated in Vogan's 1986 ICM plenary speech. The hyperbolic step and elliptic step are well understood, while the nilpotent step to construct unipotent representations in correspondence with nilpotent orbits has been extensively studied in several different perspectives over the last thirty years. Still, the final definition of unipotent representations remains to be mysterious. The aim of this talk is to show that our recent work (joint with Pandzic and Vogan) on classifying unitary representations by their Dirac cohomology shed light on what kind of irreducible unitary representations should be defined as unipotent.

11:00-12:00, N202
Chufeng Nien: Converse theorem on distinction.
Abstract: Twisted Gamma factors of a (generic or cuspidal) representation π against all generic representations τ encode information about the representation π. Converse theorem reveals how many twisted Gamma factors we should consider in order to single out a unique isomorphism class. This talk is about the relation between special values of twisted Gamma factors and $GL_n(F)$-distinguished representations of $GL_n(E)$, where E is a quadratic extension of F.

14:30-15:30, N219

Zhi Qi: Bessel identities over the complex field.
Abstract: A fundamental class of special functions in the number theory and representation theory for $\mathrm{GL}_2(\mathbb{R})$ is classical Bessel functions (over $\mathbb{R}_+$). Certain (integral) formulae for Bessel functions have deep representation theoretic interpretations. For example the formulae of Weber and Hardy on the Fourier transform of Bessel functions on $\mathbb{R}$ may be considered as a realization of the Waldspurger correspondence over $\mathbb{R}$. In this talk, we show how certain Bessel identities in the Waldspurger correspondence over $\mathbb{C}$ follow from the Weber-Hardy type formula over $\mathbb{C}$.This is a joint work with Jingsong Chai.

15:45-16:45, N219
Lei Zhang: The exterior cubic L-function of GU(6) and unitary automorphic induction
Abstract: In this talk, we will discuss an extension of Ginzburg-Rallis' integral representation for the exterior cube automorphic L-function of GL(6) to the quasi-split unitary similitude group GU(6).
Furthermore, we introduce the automorphic induction for GU(n) and show that those exterior cube L-functions have poles if and only if the cuspidal representations are automorphically induced from GU(3).

July 10, 9:15-10:15, N202
Kai Wang: An introduction to holomorphic isometries on symmetric domains.
Abstract: In this talk, we will survey some recent progress on holomorphic isometries on symmetric domains. We will construct isometric holomorphic embeddings of the unit ball into higher rank symmetric domains in an explicit way using Jordan triple systems, and prove uniqueness results for domains of rank 2, including the exceptional domain of dimension 16.

10:30-11:30, N202
Pavle Pandzic: Dirac index and twisted characters
Abstract: Dirac operators have played an important role in representation theory of real reductive Lie groups since the work of Parthasarathy and Atiyah-Schmid on the construction of discrete series representations in the 1970s.
One of the important invariants of representations is the Dirac index. An algebraic way to define the Dirac index is as the Euler characteristic of the Dirac cohomology of the associated Harish-Chandra module. The concept of Dirac cohomology was introduced by Vogan and subsequently studied by Huang-Pandzic and others. One of the important properties of the Dirac index of a representation in the equal rank case is its close relationship with the character on the compact Cartan subgroup.
In the unequal rank case, the Dirac index of all representations is zero and therefore it is a useless notion. We have however introduced a new invariant, twisted Dirac index, which is a good substitute for the classical notion in the unequal rank cases. In this lecture I will first review some basic facts about representations, Harish-Chandra modules, Dirac cohomology and index. I will then explain the notion of twisted Dirac index and present some

examples and applications. This is joint work with Dan Barbasch and Peter Trapa.

14:30-15:30, N202
Chengbo Zhu: Orbit method and unipotent representations
Abstract: A fundamental problem in representation theory is to determine the unitary dual of a given Lie group G, namely the set of equivalent classes of irreducible unitary representations of G. A principal idea, originated in a groundbreaking paper of A. A. Kirillov in the sixties, is that there is a close connection between irreducible unitary representations of G and the orbits of G on the dual of its Lie algebra. This is known as the orbit method.
In this talk, I will describe basic ideas of the orbit method as well as a recent development on the problem of unipotent representations, which is to associate unitary representations to nilpotent coadjoint orbits and which is the hardest part of the orbit method. We solve this problem for real classical groups, by combining analytic ideas of R. Howe on theta lifting and algebro-geometric ideas of D. A. Vogan, Jr. on associate varieties. Geometrically, our construction is guided by Kraft-Procesi construction of closures of nilpotent conjugacy classes in classical Lie algebras. This is joint work with J.-J. Ma and B. Sun.

15:45-16:45, N202
Dihua Jiang: Automorphic Bessel Descents and Related Problems
Abstract: The theory of endoscopic classification of the discrete automorphic spectrum of classical groups provides a fundamental structure for automorphic representations. The theory of automrphic descents is to understand the refined structure and properties of automorphic representations, based on the endoscopy theory.

In this talk, we will discuss the progress on theory of automorphic Bessel descents from my joint work with Lei Zhang and also with Baiying Liu and Bin Xu.

July 11, 9:15-10:15, N202
Jun Yu: Geometric interpretation of the Kirillov conjecture
Abstract: The Kirillov conjecture in the 1960s assers that the restriction of any irreducible unitary representation of GL(n,K) (K=R or C) to a microbolic subgroup is irreducible, which is shown by Sahi (for tempered reppresentations) and Baruch (in general) after 40 years. In this talk we give a geometric interpreation of this conjecture in the framework of Kirillov-Duflo's orbit method. This is joint work with Gang Liu (France).

10:30-11:30, N202
Sidhartha Sahi: The Capelli eigenvalue problem

Afternoon: Free discussions.

July 12, 9:15-10:15, N202
Birgit Speh: Restrictions of representations of rank one orthogonal groups : results and applications.
Abstract: I will discuss the restriction of infinite dimensional representations of the rank one orthogonal group O(n+1,1) to O(n,1) and discuss the space of symmetry breaking operators for any pair of irreducible representations of G and the subgroup G' with trivial infinitesimal character. I will discuss the application of this result to the multiplicity conjecture by B.Gross and D.Prasad for tempered principal series representations of (SO(n+1,1), SO(n,1)) with trivial infinitesimal character .These results also allow us to find periods of irreducible representations of the Lorentz group with nonzero-cohomologies. This is joint work with T. Kobayashi.

10:30-11:30, N202
Dongwen Liu: Tower property and theta lifting for loop groups
Abstract: We study the theta lifting for loop groups and extend the classical tower property established by S. Rallis to the loop setting. As an application we give the first examples of nonvanishing cusp forms on loop groups. This talk is based on a joint work with Yongchang Zhu.

14:30-15:30, N202
E. Sayag: Counting of lattice points and Harmonic Analysis
Abstract: The classic lattice counting problem in the plane (Gauss) and in the hyperbolic plane (Delsarte) insures a close relationship between the number of lattice points in a Ball of radius R and the volume of that Ball. Similar problems were studied in the 90s for symmetric spaces using Harmonic analysis (Duke-Rudnick-Sarnak) and Ergodic methods (Eskin-Mcmullen). Since then many researchers extended these results to cover other ho-

mogeneous spaces and studied the error term of the counting.
We shall give an overview of the Harmonic analysis method and then focus on obtaining effective bounds on the error term in the counting in the case of reductive spherical spaces. Our results are based on a novel comparison of Lp norms of Eigenfunctions on these spaces.
Joint work with Bernhard Kroetz and Henrik Schlichtkrull (Acta Mathematica Sinica, 2018).

15:45-16:45, N202
Zhuohui Zhang: Functions on $U(2)$ and the $(\mathfrak{g}, K)$-Module Structure of Principal Series
Abstract: I will describe the differential action of $\mathfrak{g}$ on the principal series representations of $G = SU(2,1)$ and $Sp(4,\mathbb{R})$. The structure of the principal series for G can be described explicitly by the $\mathfrak{g}$-action on the matrix coefficients of the maximal compact subgroup $K \subset G$. The computation, based on the Clebsch-Gordan coefficients of the representations of K, is algorithmic and generalizable to the real reductive groups with a maximal compact subgroup isogenous to a product of multiple copies of $SU(2)$ and $U(1)$. As an application of the machinery, I will describe some subquotients of the principal series, their restrictions to K, and I will also use the combinatorial properties of hypergeometric functions to calculate the intertwining operators of minimal principal series.

July 13, 9:00-10:00, N202

Kai-Wen Lan: Local systems over Shimura varieties: a comparison between two constructions.

Abstract: Given a Shimura variety X associated with some algebraic group G, and some algebraic representation V of G (satisfying some conditions when restricted to the center), we can define two kinds of filtered vector bundles with integrable connections, over X. The first one is based on the classical complex analytic construction using double quotients, while the second one is a new p-adic analytic construction based on the p-adic Riemann-Hilbert correspondence in the recent work by Ruochuan Liu and Xinwen Zhu. We know how to relate these two when the local systems are given by the relative cohomology of some family of abelian varieties over X. But what should we do when X is a general Shimura variety, in which case no convenient family of algebraic varieties (or, rather, "motives") are available? A priori, the complex and p-adic analytic constructions have very little in common. In this talk, we shall review the background materials and formulate the problem more precisely, and give an answer. (This is joint work with Hansheng Diao, Ruochuan Liu, and Xinwen Zhu.)

10:30-11:30, N202

Binyong Sun: Cohomologically induced distinguished representations

Abstract: Cohomological parabolic induction, which is closely related to complex analysis, provides a general method to construct irreducible representations of real reductive groups. We give a general construction of local periods on these cohomologically induced representations. Afternoon: Free discussions.